The Ex Ante Curse: Why is the Real Interest Rate so Confusing?

Onsurang Pipatchaipoom and Stefan C. Norrbin

Department of Economics, Finance, and Quantitative Analysis
Samford University

E-Leader Conference , Bangkok

January 2, 2008

What is the *Ex Ante* Real Interest Rate?

- The real expected return from holding the one-period bond from t to $t+1$.

$$r_t^e = i_t - \pi_{t+1}^e$$

- However, the expected inflation rate is unobserved in the data. Thus, the *ex ante* real interest rate is also unobservable.

- The literature approaches this problem by proposing various techniques of estimating the *ex ante* real interest rates.

Outline of the Talk

- Motivation
  - ⋆ Inconclusive results of hypothesis testing using real interest rates
  - ⋆ Does the approach of calculating the real interest rate matter?
- Methods of constructing the real interest rates
- Data and results
- Conclusions
  - ⋆ Answer: Yes. Different techniques of estimation provide different real interest rate series.
  - ⋆ The choice of data frequency, the methods of calculating inflation, and the choice of price proxies all have a significant impact on the estimated real interest rate.

Why is the Real Interest Rate Important?

- The real interest rate movement is
  - ⋆ a center element in models of business cycle
  - ⋆ a key variable in theories of exchange rate determination
  - ⋆ a valuable information for policy design
- Most theoretical models would argue that the real interest rate is stationary.
- However, there are mixed empirical findings about the stationarity of the real interest rate.

Example: Inconsistent Findings about the Real Interest Rate Parity Hypothesis

RIP Hypothesis: real interest rates should be equalized across countries.

$$r_t^e = r_t^{e*}$$

A simple test involves testing

$$r_t^e = a + \beta r_t^{e*} + \varepsilon_t$$

for $a = 0$ and β is significantly different from zero.

Support the RIP hypothesis	Reject the RIP hypothesis
Goodwin and Grennes (1994)	Cumby and Mishkin (1986)
Holme and Maghrebi (2004)	Fraser and Taylor (1990)

Does the methodology of constructing the real interest rates matter?

- Perhaps such a controversy may result from the methodology used in constructing the *ex ante* real interest rate.

- If the methodology matters, then researchers have to be more careful when selecting the approach to use.

Plan of this paper

To answer this question, we follow the following steps:

1. Survey the literature to find the commonly used methods of estimation.

2. Duplicate the techniques and use U.S. data to obtain the real interest rate from each technique.

3. Compare the time series properties of each series.

Methods of Constructing *Ex Ante* Real Interest Rates

The *ex ante* real interest rate:

$$r_t^e = i_t - \pi_{t+1}^e$$

1. ***Ex post* real rate**

$$r_t^p \equiv i_t - \pi_{t+1}$$

By assuming rational expectations, we replace π_t^e with the actual inflation such that the *expected* real rate is equal to the *realized* real rate plus inflation forecast errors

$$r_t^e = r_t^p + \varepsilon_{t+1}$$

2. **Linear regression models**

- **AR(p) specification for the expected inflation**

$$\hat{r}_t^e = i_t - \hat{\pi}_{t+1}$$

where $\hat{\pi}_{t+1} = \hat{\varphi}_1 \pi_{t-1} + \hat{\varphi}_2 \pi_{t-2} + ... + \hat{\varphi}_p \pi_{t-p}$

***Intuition**:* Agents use the past behavior of the inflation rate to form expectations of the next period rate of inflation.

- **Mishkin's Linear Projection**

  Mishkin used a set of observable variables, X_t to linearly project r_t^e into X_t as

$$P(r_t^e | X_t) = X_t \beta$$

  Step 1: Estimate $\hat{\beta}_{OLS}$ from $r_t^p = X_t \beta + u_t - \varepsilon_{t+1}$
  Step 2: Construct $\hat{r_t^e} = X_t \hat{\beta}_{OLS}$

  The choice of X_t are 4 lags of the inflation rate, 1 lag of M1 growth rate, the nominal interest rate, and the forth order polynomial in time.

  ***Intuition**:* Agents use all available information (inflation rate and other macro variables) at time t to project the real interest rate.

- **Recursive Least Squares**

  We estimate the expected inflation rate (π^e_{t+1}) by obtaining the one-period ahead forecast of

$$\Phi(L)\pi_t = \Theta(L)\varepsilon_t$$

  where $t = 1, ..., k, k+1, ..., T$.

  Start with $t = 1, ..., k$ for the 1st iteration and then add on more observation to the model until all data are used up

- **Rolling Regression**
  Same as the RLS approach, but instead of adding more observations to each iteration, we fix the time interval moving window.

Intuition: Agents frequently update their inflation forecasts once they learn about changes of inflation rate in the past.

3. **Nonlinear Approaches: Regime-Switching Model**

The 3-state Markov-switching mean-variance model

$$(r_t^p - \mu_{s_t}) = \phi_1(r_{t-1}^p - \mu_{s_{t-1}}) + \phi_2(r_{t-2}^p - \mu_{s_{t-2}}) + e_t$$

where

$$e_t \sim N(o, \sigma_{s_t}^2)$$

$$\mu_{s_t} = \mu_1 S_{1t} + \mu_2 S_{2t} + \mu_3 S_{3t}$$

$$\sigma_{st}^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} + \sigma_3^2 S_{3t}$$

$S_{jt} = 1,$ if $S_t = j$ and $S_{jt} = 0,$ otherwise $j = 1, 2, 3$

$$p_{ij} = Pr[S_t = j | S_{t-1} = i], \ \sum_{j=1}^{3} p_{ij} = 1$$

The *ex ante* real interest rate can be estimated as:

$$E(r_t^p|\tilde{r}_{t-1}^p) = E(\mu_{S_t}|\tilde{r}_t^p)+\phi_1 E(r_{t-1}^p-\mu_{s_{t-1}}|\tilde{r}_t^p)+\phi_2 E(r_{t-2}^p-\mu_{s_{t-2}}|\tilde{r}_t^p)$$

where $\tilde{r}_t^p = [r_1^p ... r_t^p]$

We use the Gibbs-sampling technique to obtain the distribution and parameter estimates.

Intuition: Agents take the possibilities of structural changes in the real interest rate into account. In particular, they allow the mean and variance of the real interest rate to shift between 3 different regimes (low, medium, and high).

Data

- Monthly (1971M1-2003M12)
  - ⋆ Nominal interest rate: 1-month Eurodollar deposit rate (London)
  - ⋆ CPI
- Quarterly (1971Q1-2003Q4)
  - ⋆ Nominal interest rate: 3-month Treasury bill rate
  - ⋆ CPI

Inflation Calculations

- The period-to-period annualized inflation rate
  - ⋆ $\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)^{12}$ for monthly data
  - ⋆ $\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)^{4}$ for quarterly data
- The year-to-year annualized inflation rate
  - ⋆ $\pi_t = \ln\left(\frac{P_t}{P_{t-12}}\right)$ for monthly data
  - ⋆ $\pi_t = \ln\left(\frac{P_t}{P_{t-4}}\right)$ for quarterly data

The Constructed Real Interest Rates

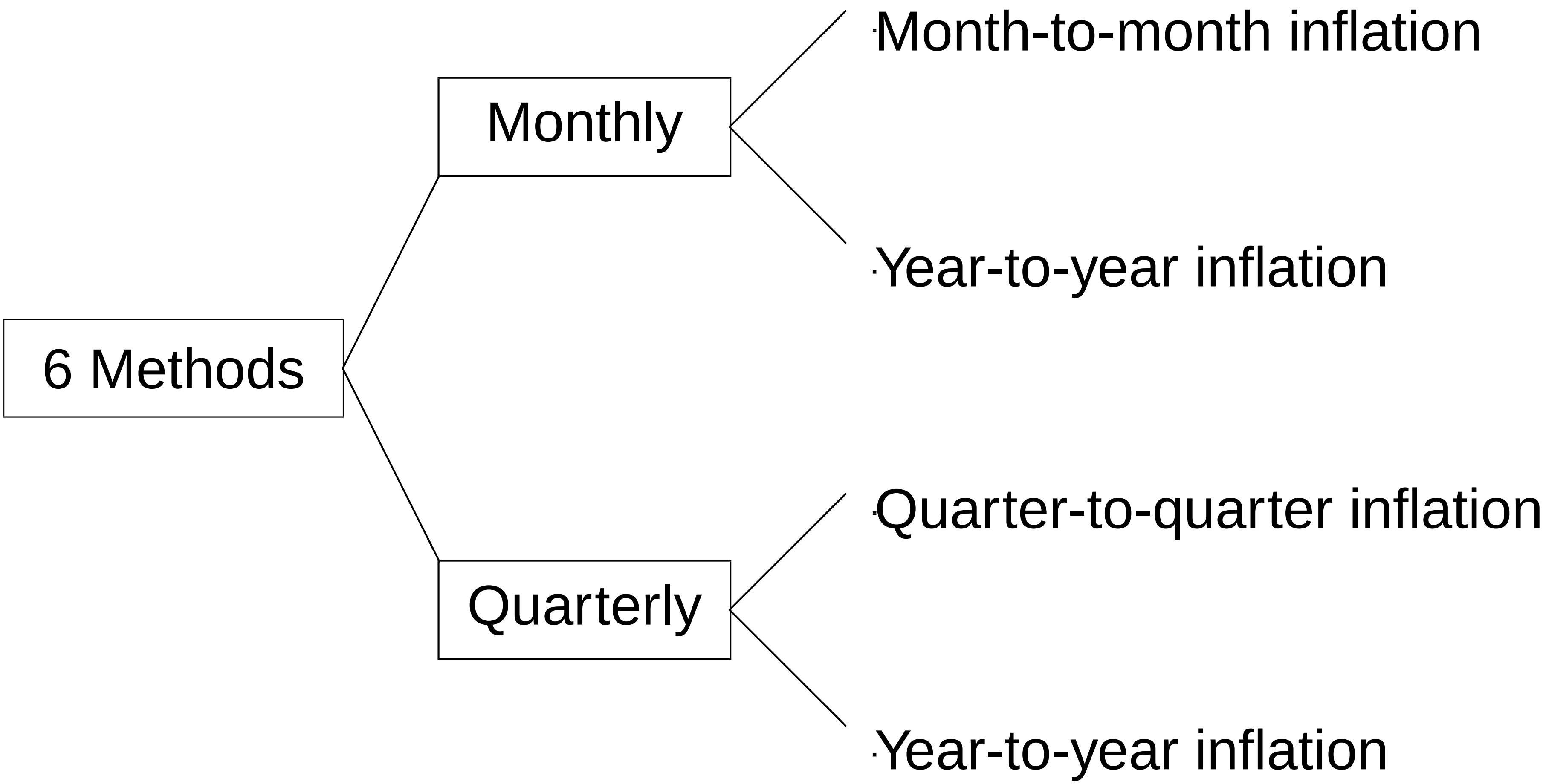
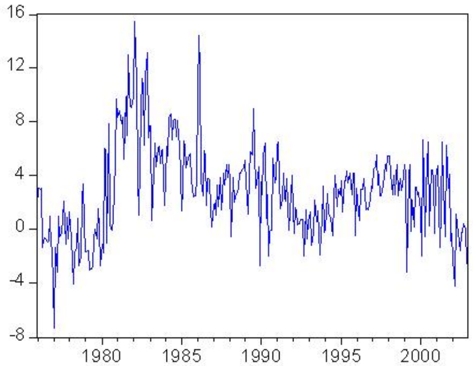
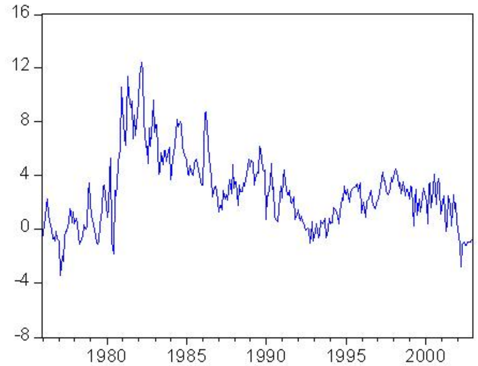


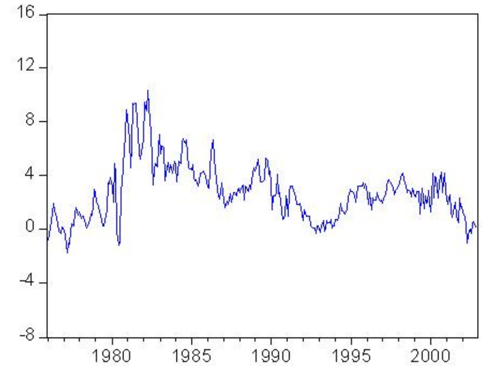
Figure 1: Real Interest Rates: Using Monthly Data and Month-to-Month Inflation Calculation



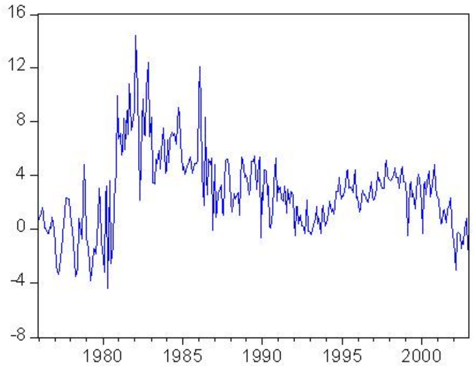
Ex Post Real Interest Rate



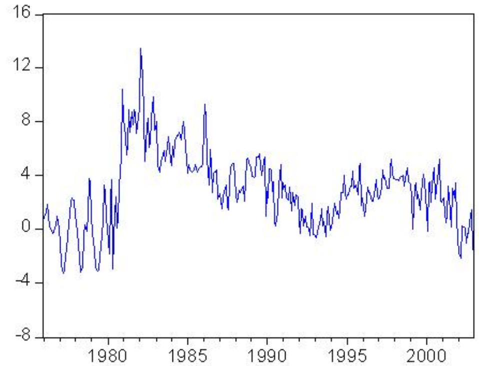
Ex Ante Real Interest Rate: AR(4)



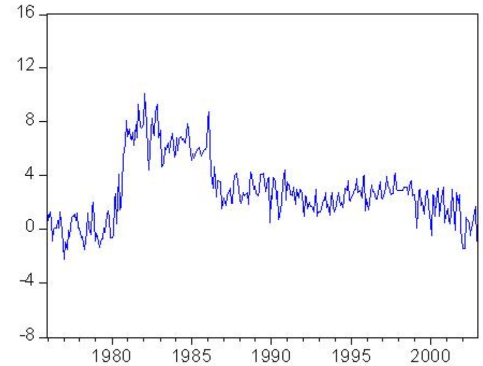
Ex Ante Real Interest Rate: Mishkin



Ex Ante Real Interest Rate: Rolling Regression



Ex Ante Real Interest Rate: Recursive Least Squares



Ex Ante Real Interest Rate: Regime-Switching Model

Figure 2: Real Interest Rates: Using Monthly Data and Year-to-Year Inflation Calculation

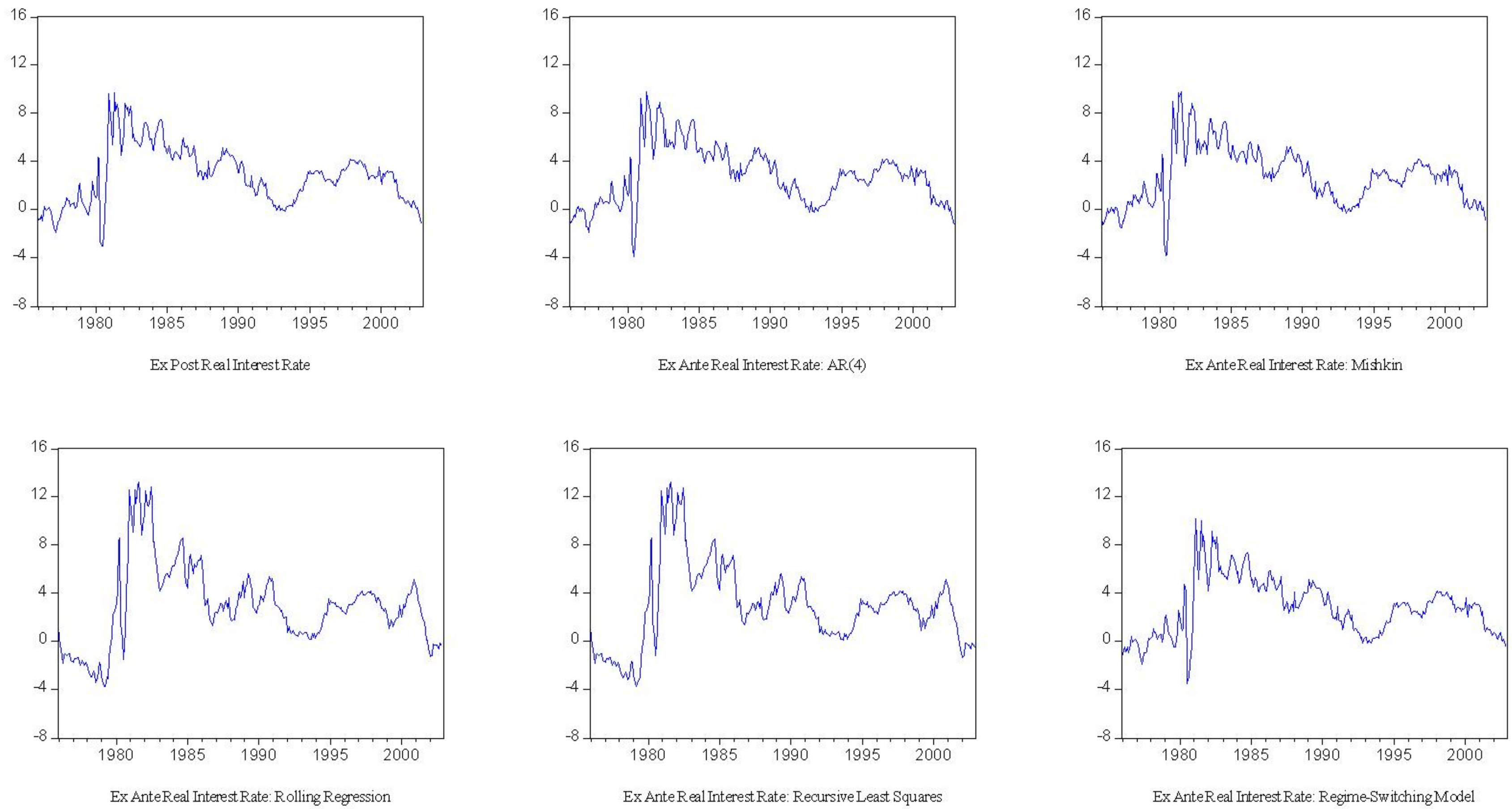


Figure 3: Real Interest Rates: Using Quarterly Data and Quarter-to-Quarter Inflation Calculation

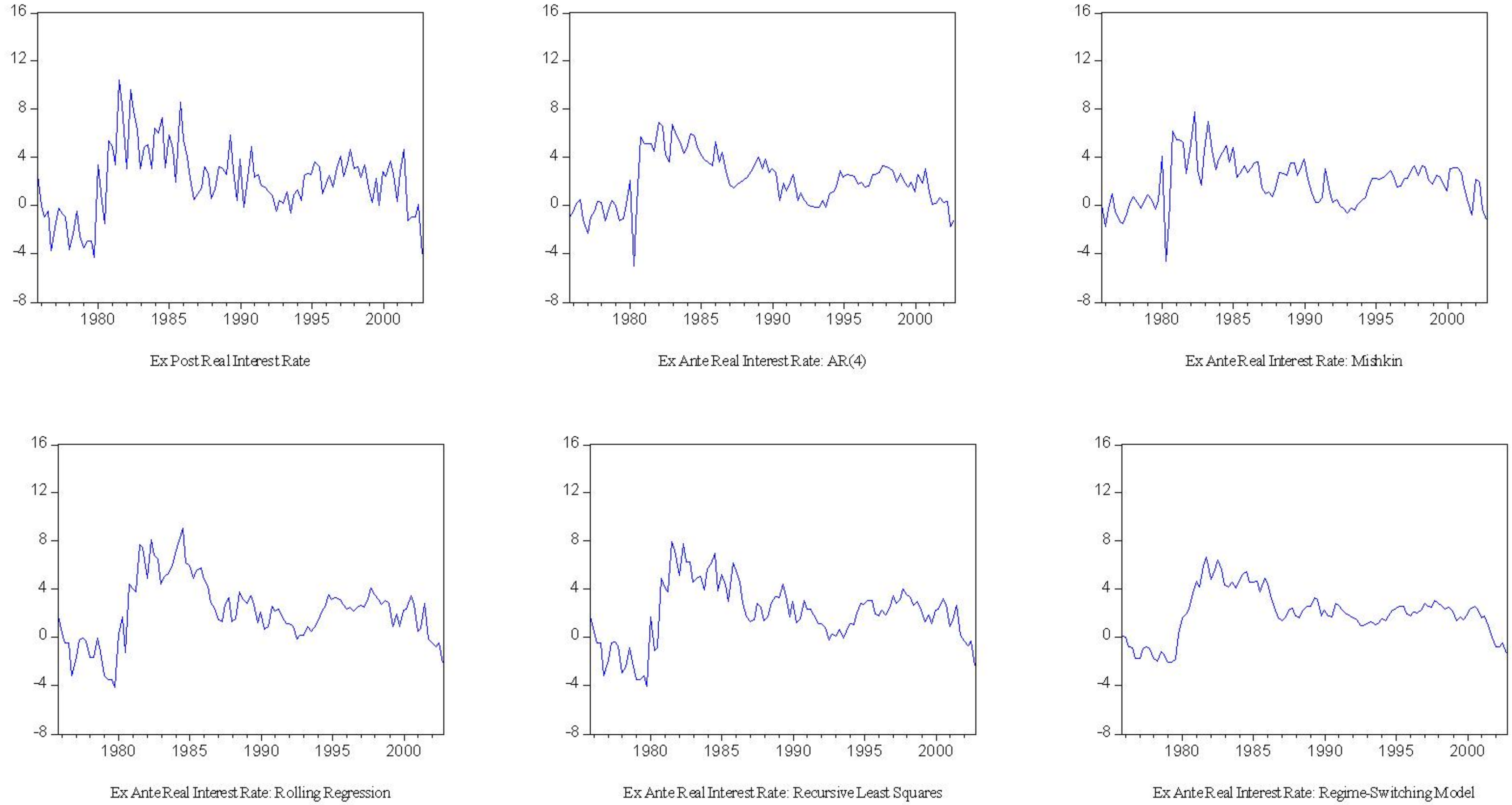
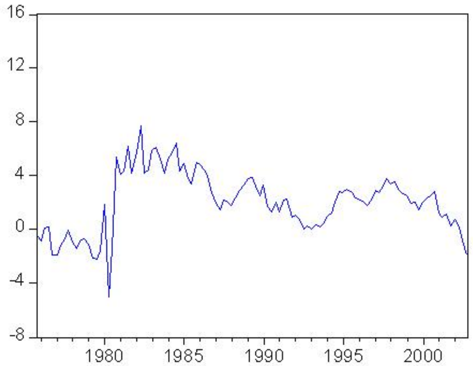
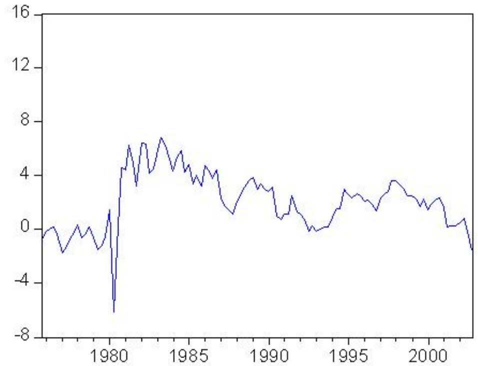


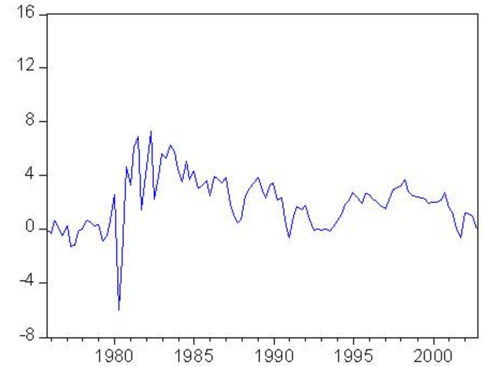
Figure 4: Real Interest Rates: Using Quarterly Data and Year-to-Year Inflation Calculation



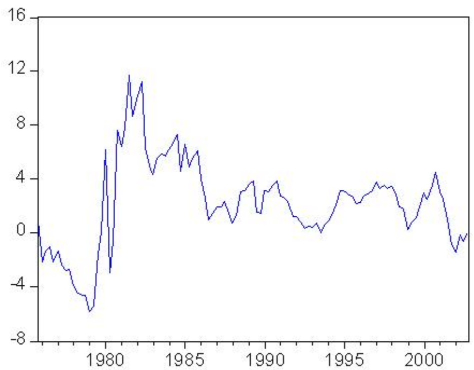
Ex Post Real Interest Rate



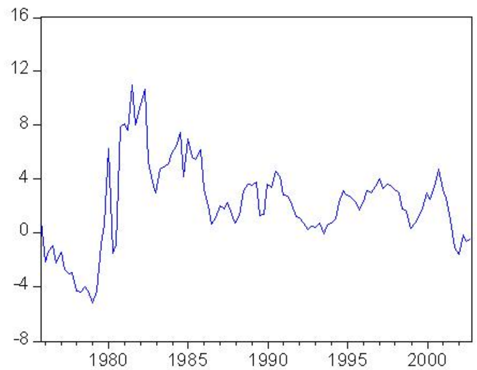
Ex Ante Real Interest Rate: AR(4)



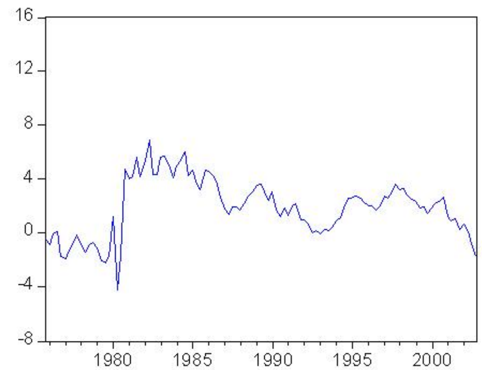
Ex Ante Real Interest Rate: Mishkin



Ex Ante Real Interest Rate: Rolling Regression



Ex Ante Real Interest Rate: Recursive Least Squares



Ex Ante Real Interest Rate: Regime-Switching Model

Table 1: Real Interest Rates: Monthly Data and Month-to-Month Inflation Calculation

Statistics	Ex Post	AR(4)	Mishkin	Rolling	RLS	Regime
Mean	2.904	2.845	2.757	2.859	2.923	2.898
Median	2.938	2.661	2.690	2.676	2.779	2.616
Maximum	15.520	12.448	10.320	14.425	13.434	10.129
Minimum	-7.333	-3.391	-1.742	-4.320	-3.256	-2.171
Standard dev.	3.398	2.718	2.111	2.955	2.753	2.381

Table 2: Correlations Between Real Interest Rates: Monthly Data and Month-to-Month Inflation Calculation

	Ex Post	AR(4)	Mishkin	RLS	Rolling	Switching
Ex Post	1.000	0.703	0.625	0.883	0.895	0.668
AR(4)		1.000	0.918	0.880	0.798	0.838
Mishkin			1.000	0.815	0.731	0.810
RLS				1.000	0.966	0.800
Rolling					1.000	0.758
Switching						1.000

Table 3: Tests for Equality of Means and Variance between Real Interest Rates

	Real interest rates from different approaches			
	Monthly		Quarterly	
	$\ln(\frac{p_t}{p_{t-1}})^{12}$	$\ln(\frac{p_t}{p_{t-12}})$	$\ln(\frac{p_t}{p_{t-1}})^{4}$	$\ln(\frac{p_t}{p_{t-4}})$
Mean Equality Test:				
Test Statistics	0.152	0.294	0.171	0.209
	(0.980)	(0.943)	(0.973)	(0.959)
Variance Equality Test:				
Test Statistics	13.238	9.617	3.516	6.456
	(0.000)	(0.000)	(0.000)	(0.000)

Stationarity vs. Nonstationarity

Consider the AR(1) model,

$$y_t = \varphi y_{t-1} + \varepsilon_t$$

- If $\varphi = 1$, then y_t is a pure random walk, or *nonstationary* $I(1)$ such that $y_t = y_{t-1} + \varepsilon_t$ and

$$\Delta y_t = \varepsilon_t$$

- If $|\varphi| < 1$, then y_t is mean reverting, or *stationary* $I(0)$.

Unit Root Tests

- The null hypothesis is $I(1)$

  ⋆ Conventional tests

  1. Augmented Dickey-Fuller (ADF) test
  2. Phillips-Perron (PP) test

  ⋆ Recent tests

  1. Elliot, Rothenberg and Stock (DF-GLS) test
  2. Ng-Perron test

Table 4: ADF Unit Root Test: Results of the Monthly and Quarterly Data

Tests	Ex Post	AR(4)	Mishkin	Rolling	RLS	Regime
			Monthly Data			
Month-to-month inflation	$I(0)$	$I(1)$	$I(1)$	$I(0)$	$I(0)$	$I(1)$
Year-to-year inflation	$I(1)$	$I(1)$	$I(0)$	$I(0)$	$I(0)$	$I(1)$
			Quarterly Data			
Quarter-to-quarter inflation	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
Year-to-year inflation	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(0)$	$I(1)$

Conclusions

- The *ex ante* real interest rates constructed by different approaches do follow different time series processes.

- The stationarity conclusions of the real interest rates are sensitive to

  ⋆ the choice of methods
  ⋆ the sampling frequency
  ⋆ the computations of the inflation rate